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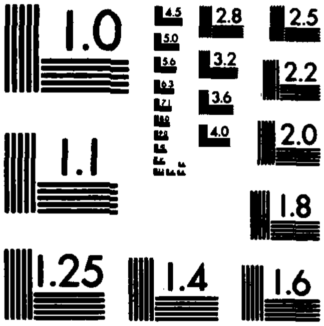

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The objective of the research has been to develop engineering methodologies applicable, but not limited, to aerospace automatic control design problems in which there are performance specifications requiring precise control of system behavior in the presence of stochastic disturbances (e.g., wind gusts) and large-but-bounded uncertainties in the dynamical response of the system (e.g., parameter uncertainty, unmodeled nonlinearities, and so forth). During the past three years of AFOSR supported research, a cohesive body of theory has been developed that enables engineers to relate the ability of feedback control systems to meet such specifications directly and quantitatively to the "return difference matrix" associated with the system's feedback loops. Now results enabling "L^∞ optimization" of returned difference singular value Bode plots promise to be of great value in robust multivariable feedback controller synthesis. Continuing research is currently being aimed at further tightening the links between this theory and the most recent developments of modern stochastic linear optimal control synthesis (CONTINUED)

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ITEM #19, CONTINUED: theory, and extending the results to admit more practical problems, so that this theory may be used more effectively by engineers to efficiently and systematically design the feedback gains that determine a feedback system's return difference matrix. Such results substantially reduce the dependence of control engineers on intuition, simulation, and luck and provide the know-how to successfully and efficiently solve the increasingly complex and demanding aerospace control problems of the coming decades.

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RETURN DIFFERENCE FEEDBACK DESIGN FOR ROBUST

UNCERTAINTY TOLERANCE IN STOCHASTIC MULTIVARIABLE

CONTROL SYSTEMS, INTERIM REPORT, GRANT AFOSR-80-0013

1 OCT 81 - 30 SEP 82

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Contents

ABSTRACT

The objective of the research has been to develop engineering methodologies applicable, but not limited, to aerospace automatic control design problems in which there are performance specifications requiring precise control of system behavior in the presence of stochastic disturbances (e.g., wind gusts) and large-but-bounded uncertainties in the dynamical response of the system (e.g., parameter uncertainty, unmodeled nonlinearities, and so forth). During the past three years of AFOSR supported research. A cohesive body of theory has been developed that enables engineers to relate the ability of feedback control systems to meet such specifications *directly and quantitatively* to the "return difference matrix" associated with the system's feedback loops. Now results enabling "L^{∞} optimization" of returned difference singular value Bode plots promise to be of great value in robust multivariable feedback controller synthesis. Continuing research is currently being aimed at further tightening the links between this theory and the most recent developments of modern stochastic linear optimal control synthesis thoery, and extending the results to admit more practical problems, so that this thoery may be used more effectively by engineers to efficiently and systematically design the feedback gains that determine a feedback system's return difference matrix. Such results substantially reduce the dependence of control engineers on intuition, simulation, and luck and provide the know-how to successfully and efficiently solve

the increasingly complex and demanding aerospace control problems of the coming decades.

INTRODUCTION

The increased demands for quick and precise control over aircraft and space vehicle response that are anticipated in the coming decades will have to be matched with automatic control systems that can respond instantaneously, without moment-to-moment human guidance, anticipating vehicle response insofar as is possible and, more importantly, continually and automatically monitor vehicle response and re-adjust control signals to correct for unpredicted deviations from the desired response. Such unpredicted response variations can result from external disturbances (e.g., wind gusts) and from the impossibility of employing a sufficiently complex and accurate model of the vehicle's dynamics to account for every vibrational mode, every nonlinearity, ..., every variable affecting system response.

While the state-space-based mathematical theory for controlling systems without substantial uncertainty regarding dynamical response grew relatively sophisticated during the two decades of the 1960's and 1970's, there was almost no significant progress concerning the control of systems having uncertain response since the 1940's and 1950's when great strides were made in the development of the "classical" transfer-function-based theory for the control of simple single-input-single-output (SISO) linear time-invariant (LTI) systems with uncertainty. Consequently, when this research project was begun in October 1979 there was no adequate theory to provide engineers with an effi-

cient, systematic procedure for the design of precision controllers for more complex multi-input-multi-output systems such as the highly unstable, fast responding, control confifured aerospace vehicles that are expected to be operating in tomorrow's combat environment. The objective of the present research has been to develop this badly needed theory so that the engineers who must design tomorrow's aerospace vehicles will have more than intuition, trial-and-error simulation and mystical "seat-of-the-pants" insight for guidance in designing uncertainty-tolerant automatic controllers for these vehicles.

BACKGROUND AND PROGRESS

Since work began on this project in October 1979, the research effort has been generally successful in achieving its main objective of relating the return difference matrix to the uncertainty tolerance properties of a system. These properties are also known as *robustness* properties or *feedback* properties. Our results, together with some related results useful in the actual synthesis of robustly uncertainty tolerant feedback controllers, were reported in the paper "Feedback Properties of Multivariable Systems: The Role and Use of the Return Difference Matrix" [1]. This paper discusses the central roles in feedback theory of the return difference matrix (denoted $I+L(s)$) and the inverse-return difference matrix (denoted $I+L^{-1}(s)$).

Among the new theoretical results in [1] are the following:

(i) A new method for exact evaluation of the sensitivity of multivariable feedback control systems which overcomes significant practical limitations associated with previously known methods. *Sensitivity to large plant and sensor variations is directly related to the nominal system's return and inverse-return difference matrices.* (See Theorem 2.1 and Theorem 2.2 in [1]).

(ii) Significant drawbacks of characteristic locus analysis methods (cf. MacFarlane and others) are described. Return and inverse-return difference singular value plots are found to overcome some of the drawbacks of characteristics loci. The results have been found to be useful in quantifying some fundamental limits on the achievable performance of feedback control

systems. (See Section 3 and Section 4 of [1]).

(iii) A technique, based on stochastic linear quadratic Gaussian (LQG) optimal control theory, has been developed to aid the shaping of the return and inverse-return difference singular value plots. Though the technique is to a certain extent a trial-and-error design technique, it continues to be substantially more systematic than any other method that is currently available for synthesizing multivariable control systems to meet specifications requiring a robust tolerance of disturbances, noise and plant/sensor modeling errors. (See Section 5 in [1].) To demonstrate the viability of the of the technique, the theory has been applied to the synthesis of an automatic controller for the longitudinal dynamics of an advanced control configured vehicle (CCV) aircraft, viz., the NASA HIMAT remotely piloted aircraft (see Section 6 of [1]).

More recently research effort has focused on several issues.

First, substantial effort has been focused on the important practical issue of how to solve the so-called "Inverse Problem of Linear Quadratic Gaussian (LQG) Optimal Control" in the general setting in which the controller is dynamical and the plant is subject to plant and sensor noise. This inverse problem is as follows: Given a realizable closed-loop control return-difference matrix, the plant transfer function matrix $P(s)$, and the plant and sensor noise power spectra matrices, say $\Sigma_d(s)$ and $\Sigma_n(s)$, find the linear quadratic cost matrices $R(s)$ and $Q(s)$ such that the closed-loop system is optimal in the sense

that the following stochastic cost is minimized;

$$J = \frac{1}{2\pi j}\int_{-j\infty}^{+j\infty} \mathrm{Tr}\left(Q(s)\Sigma_y(s) + R(s)\ \Sigma_u(s)\right) ds$$

where $\Sigma_y(s)$ and $\Sigma_u(s)$ denote the respective closed-loop power spectra matrices of the sensor output and control input signals. We are pleased to report that the solution to the problem, together with extensive discussion of its ramifications regarding a number of related problems is in hand [5]. This work was supported under AFOSR Grant 80-0013.

The importance of the LQG inverse problem to the proper understanding of robust multivariable feedback control system design cannot be understated. LQG multivariable feedback designs are preferred for a variety of reasons: Good computer software is readily available for solving the LQG design equations (e.g., [10]), LQG designs optimize inherent trade-offs between robustness properties such as sensitivity versus stability margin [1], and many engineers in the aerospace industry are familiar with the basic LQG concepts. The solution to the inverse problem plays a vital role in our understanding of how one can use the LQG theory to place the poles and zeros of I+L(s) in order to achieve acceptable robustness and sensitivity singular-value Bode plots [1] and acceptable transient response and asymptotic tracking properties [5]. Also, as it is common practice to iteratively adjust the LQG cost matrices when "fine-tuning" a control design to meet various robustness

specifications, the solution to the inverse problem provides a starting point for fine-tuning non-LQG multivariable feedback designs.

A second focus of the research effort is one of rather fundamental significance in linear feedback theory: realizability. A closed-loop feedback system's return-difference $I + L(s)$ is said to be <u>realizable</u> for a given plant $P(s)$ if for some controller $C(s)$

(i) $L(s) = P(s)C(s)$, and

(ii) $(I + L(s))^{-1}$ is stable, and

(iii) $P(s)$ and $C(s)$ are "coprime" in the sense that there are no unstable pole-zero cancellations in the product $P(s)C(s)$. A new simplified realizability result (Lemma 1, [4]) has been developed which characterizes realizability directly in terms of the poles and associated residues of the return-difference and inverse-return difference. This is an improvement over previous multivariable results([3], Lemma 3) which require a solution of the so-called "Bezout" equation and give only limited insight into the constraints imposed by realizability on the set of achievable return difference and inverse-return difference matrices.

The realizability question arose in connection with the previously mentioned LQG inverse problem, and our new realizability result in [4] plays a crucial role in the solution of the LQG inverse problem in [5], in addition to laying the groundwork for stability margin optimization problem solved in [4].

The problem of stability margin optimization and the mathematically dual problem of sensitivity minimzation with respect to the L^∞ norm have been a third major thrust of our research effort in the past year. With the aid of results in [1] which show that stability margin is inversely proportional to the L^∞ norm of the inverse of the inverse-return difference matrix $I + L^{-1}(s)$ and the aid of our new realizability result (Lemma 1 of [4]), it is shown in [4] that the problem of designing a feedback controller to maximize stability margin (subject to decoupling and, perhaps, asymptotic tracking constraints) is mathematically equivalent to the minimization

$$\min_{x_j(s)\in H^\infty} \|x_j(s)\|_\infty \quad , \qquad (j = 1, \ldots., m)$$

subject to complex interpolation constraints of the form

$$x_j(s_{ij}) = w_{ij}, \qquad (i = 1, \ldots, n),\ \mathrm{Re}(s_i) > 0,$$

where s_{ij} and w_{ij} are complex constants and H^∞ is the Hardy space of stable transfer functions with the L^∞ norm,

$$\|x(s)\|_\infty \triangleq \sup_\omega |x(j\omega)|.$$

A simple solution to this interpolation problem requiring only the calculation of certain eigenvectors and eigenvalues is available in the mathematics literature [39]. This leads in turn to the multivariable feedback controller having maximal stability robustness. It also points the way to improving

and extending to multivariable systems certain recent results of Zames and Frances [40] concerning single-loop feedback sensitivity minimization with respect to the L^∞ norm. The results recently have been generalized to the decoupled multivariable case by the principle investigator and Ph.D student B.S. Chen [4,5].

Another area which we have studied during the past year has been the use of the so-called Hankel singular values [8-15] as an alternative measure of the "sizes" of the return difference $I + L(s)$ and inverse return difference $I + L^{-1}(s)$ for purposes of robustness continuation proposal, the Hankel singular values are attractive for at least two reasons:

(1) There are finitely many Hankel singular values for a finite order system whereas conventional frequency response singular values involve an infinite continuum of ω-dependent singular values (several at each frequency $\omega \in (-\infty, \infty)$). It can be shown that this discrepancy stems from the following fundamental mathematical fact - the *Hankel* operator is *compact* with a finite, discrete spectrum (i.e., set of eigenvalues), whereas the frequency-response method corresponds to *noncompact Toeplitz* operator with a *continuous* spectrum (with eigenvalues equal to the set eigenvalues of the operators frequency response matrix as frequency ω ranges over all real values $\omega \in (-\infty, \infty)$).

(2) Hankel singular value decomposition of a linear system captures the causal nature of a system and is closely related to well-understood state-space concepts such a controllability and observability. In contrast frequency response singular values do not enable one to readily distinguish realizable (i.e., causal) from non-realizable (i.e., non-causal systems.

On the encouraging side, Hankel singular values have been found to correlate well with the practically relevant time-response and frequency-response properties of lightly damped aerospace structures [14] . On the other hand, we have found that one can synthesize classroom examples of transfer function pertubations which have vanishingly small Hankel singular values, yet are capable of destabilizing the most robust feedback design; i.e., Hankel singular values seem to induce an inappropriate topology in the space of transfer matrices [15] .

Still there are close links between Hankel singular value optimization problems and L^{∞} norm frequency response robustness optimization problems involving the return difference and inverse-return difference. The stability-constrained L^{∞} optimal complex interpolation problems that yield to the solution of sensitivity minimization problems [40] and stability margin maximization problems [39] also provide the solution to related Hankel singular value optimization problems involving reduced order system modeling (See [9] and [41]). Thus, although we do not presently see any future for the Hankel singular values as a measure of robustness, it is clear now that concepts and theoretical results developed in conjunction with optimal Hankel model reduction theory will be useful to us in performing the realizability-constrained L^{∞} - norm frecuency-response singular value optimizations that naturally arise in synthesizing optimally robust multivariable feedback controller designs.

CURRENT RESEARCH

The primary focus of our current research is on the synthesis of optimally robustly uncertainty-tolerant multivariable feedback control systems. The results presently in hand for this purpose include the L^2-norm LQG optimal synthesis [1, Section 5], L^∞-norm optimal sensitivity synthesis [40], and L^∞-norm optimal stability margin synthesis [4]. The LQG approach developed in [1] is at present the most useful of the three because it allows one to establish an optimal trade-off between the usually conflicting robustness requirements of sensitivity versus stability margin. One's choice of LQG cost/noise weighting matrices allows one to specify in each instance how much importance is to be attached to each of these two conflicting design requirements. However the L^2-norm which is the basis for LQG optimization is inappropriate for stability margin analysis since even plant perturbations $\Delta_p(s)$ having vanishingly small L^2-norm,

$$\|\Delta_p(s)\|_{L_2} \triangleq \left(\frac{1}{2\pi j}\int_{-\infty}^{\infty} \mathrm{Tr}(\Delta^{-T}(j\omega)\Delta(j\omega))\, d\omega\right)^{\frac{1}{2}},$$

can cause instability; i.e., the L_2 norm like the Hankel-norm [15], gives an inappropriate topology to the space or transfer

matrices for stability margin analysis. For stability margin analysis the L^∞ norm associated with frequency response singular values is known to be much more appropriate [1, 17, 42, 43]. For sensitivity optimization the advantages of the L^∞ norm over the L^2 norm are not so clear-cut, although Zames [44] has put forth some compelling arguments in favor of the L^∞ norm for sensitivity optimization.

However, the existing L^∞ norm robustness optimization results in [4] and [40] are inadequate for several reasons. First, the problem formulations use <u>only</u> sensitivity or <u>only</u> stability margin as a performance measure, making no provision for the trade-offs between the two which are the salient feature of most practical feedback designs. Also, the L^∞-norm sensitivity optimization results [40] are limited to single--loop feedback systems. The L^∞- norm stability margin results [4] are only slightly better, being limited to decoupled (i.e., diagonal) multiloop closed-loop transfer matrices. Additionally, for both L^∞-norm sensitivity optimization and for L^∞-norm stability margin optimization, the solutions in [4] and [40] become degenerate and fail to exist in the important case where the open-loop plant has poles or has zeros on the imaginary axis. For example, "type-1" systems (i.e., with integral feedback) are beyond the scope of the existing stability margin opmization results. Of these limitations, perhaps the failure to allow trade-off between minimal L^∞-norm sensitivity and

maximal L -norm stability margin is the most serious: virtually all practical design problems require trade-offs between sensitivity and stability margin. It is our expectation that theoretical links between this problem and the superficially unrelated but mathematically similar problems of complex interpolation and optimal Hankel model reduction [9,41,47] will prove useful in this effort.

This avenue of research proved to be spectacularly successful this year when we solved the multivariable L^∞-norm sensitivity optimization problem using this approach. The results are briefly summarized in [47], and a more detailed paper is currently being prepared for presentation at the June 1983 American Control Conference. We are quite pleased with this success because the multivariable problem had resisted the efforts of top-notch researchers for several years [40, 44].

Perhaps more importantly, our solution to the multivariable L^∞ sensitivity optimization problem may well open the door for the optimal solution of realistic L^∞ singular value shaping problems involving optimal trade-offs between stability margin singular values and sensitivity singular values. Current research is focusing on this important problem. In our opinion success in this endeavor would lead to a multivariable design tool that would have significant advantages over the rather imprecise currently available techniques, e.g., it would be preferred over linear quadratic optimal control methods for most applications.

With as primary motivation the robustness issues, we are looking more carefully at all of the norms relevant to linear systems, their interrelationships, their connections with the Hankel and the Toeplitz operators associated with the return difference and the inverse-return difference, and the topology the norms induce in th space of transfer matrices. Inspiring ourselves from [46], it seems that there are several norms relevant to linear systems; except for the L_2 norm associated with LQG theory, each of these norms is the classical spectral norm of a Hilbert space operator associated with either the system impulse response or the system transfer matrix. Examples of such operators are the Hankel operator and the Toeplitz operator, the former being related to the Hankel norm and the latter being related to the frequency response norm. Actually, as shown in [46], there are myriads of relevant operators somewhere between the Hankel and the Toeplitz structures, and, consequently, there are myriads of norms between the Hankel norm and the frequency--response norm; one is tempted to speculate that the topology becomes stronger and stronger as we proceed from the Hankel norm

In efforts to provide a more solid link between input-output and state-space measures of robustness, we have recently re-examined linear quadratic optimal control theory in the context of Hankel-Norm balanced state-space coordinates. The optimal LQG compensator is well known to consist of the cascade of the optimal filter

$$\dot{\hat{x}} = A\hat{x} + Bu + r_m^{-1}P_mC^T(z-C\hat{x})$$

and the optimal control gain

$$u = -\, r_o^{-1}\, B^TP_o\, \hat{x}\ .$$

P_m and P_o are the stabilizing, positive definite solutions to the *filtering and the control algebraic Riccati equations:*

$$AP_m + P_mA^T + q_mBB^T - P_mC^Tr_m^{-1}CP_m = 0,$$

$$A^TP_o + P_oA + q_oC^TC - P_oBr_o^{-1}B^TP_o = 0.$$

Now, a "balancing" operation

$$(A,B,C) \xrightarrow{T} (TAT^{-1},\ TB,\ CT^{-1}) =: (\underline{A},\ \underline{B},\ \underline{C})$$

is utilized to simultaneously diagonalize the solutions to the filtering and control Riccati equations:

$$\left.\begin{array}{l} P_m \xrightarrow{T} TP_mT^T \\ P_o \xrightarrow{T} T^{-T}P_mT^{-1} \end{array}\right\} = \begin{bmatrix} \mu_1 & & 0 \\ & \mu_2 & \\ & & \ddots \\ 0 & & \mu_n \end{bmatrix}$$

Little though shows that the μ's are *intrinsic quantities, independent of the original state space realization (A,B,C).*

The rationale for "balancing" the plant (A,B,C) so as to simultaneously diagonalize the solutions to the Riccati equations is that the so-defined μ's provide "measures" of the "importance" of each state component $\underline{x}_k$ of the plant G(s) in its balanced realization $(\underline{A},\underline{B},\underline{C})$. This is easily seen using the error-$(x-\hat{x})$-covariance interpretation of $\underline{P}_m = \mathrm{diag}\{\mu_1,\ldots,\mu_n\}$ and the cost interpretation of $\underline{P}_o = \mathrm{diag}\{\mu_1,\ldots,\mu_n\}$.

The theory of this Riccati balancing is fairly well developed. Probably the most striking result is the fact that deleting the compensator state components $\hat{\underline{x}}_k$'s corresponding to the lowest μ_k's leads to a *stabilizing, reduced-order* (LQG) *compensator.*

Within the scope of the present research project, which is primarily concerned with robustness and sensitivity issues in feedback design, we have investigated the *robustness aspects* of the above described reduced-order compensator design scheme.

Clearly, μ_k defines how much the (balanced) state component $\underline{x}_k$ is involved in the closed-loop LQG behavior of the system. (In particular, if μ_k is small, then $\hat{\underline{x}}_k$ can be removed in the compensator, yet the design remains stable.) *But how is μ_k affected by the robustness requirement?* The robustness properties of the LQG loop depend strongly on the parameters q and r. In particular, it is fairly well known that in some singular situations the stability margin is boosted. We have utilized the tetrahedral truss of Draper Lab as a test bed to determine how the μ's are affected by the robustness requirements on the LQG loop. This research was motivated by an attempt to derive some "rules of the thumb" to determine which state coordinates of a large space structure are the troublemakers and should be taken into account for compensation. The results are reported in paper [48]. A fundamental result is that in most cases the balanced coordinates are lined up with the

modal coordinates. *Thus the μ's can be interpreted as the importance of the corresponding vibration modes.* We have observed that the μ's of the vibration modes do depend on the robustness requirements of the loop. In general, as the stability margin requirement increases, the μ's increase and the modes are more and more difficult to discriminate importance wise. In case of modes with overlapping eigen-frequencies the μ's increase, as a warning that a resonance can occur. Finally, in case of unstably interacting oscillators (in contrast to *stably interacting oscillators, unstably interacting oscillators* can go 180 phase shift under arbitrarily small perturbations of the eigenfrequencies), the μ's are larger than for stably interacting oscillators, as a warning that instability can occur.

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